Measuring the Cosmological Lepton Asymmetry through the CMB Anisotropy

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A large lepton asymmetry in the Universe is still a viable possibility and leads to many interesting phenomena such as gauge symmetry nonrestoration at high temperature. We show that a large lepton asymmetry changes the predicted cosmic microwave background (CMB) anisotropy and that any degeneracy in the relic neutrino sea will be measured to a precision of 1% or better when the CMB anisotropy is measured at the accuracy expected to result from the planned satellite missions MAP and Planck. In fact, the current measurements already put an upper limit on the lepton asymmetry of the Universe which is stronger than the one coming from considerations of primordial nucleosynthesis and structure formation.

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1. Introduction For all we know, the Universe may contain background charges which are comparable to, if not larger than, the entropy density. The presence of some sizeable charge asymmetry may postpone symmetry restoration in nonsupersymmetric theories [1] or – even more remarkably – it can lead to symmetry breaking of internal symmetries at high temperature T [2]. Furthermore, the phenomenon of symmetry nonrestoration at high T in presence of large charge asymmetries has been recently shown to work in supersymmetry too [3].

The principal candidate for a large charge is the lepton number which today could reside in the form of neutrinos. This has inspired Linde in his original work to point out that large enough lepton number of the Universe would imply the nonrestoration of symmetry even in the Standard Model (SM) [4]. The fact that the large lepton number can be consistent with the small baryon number asymmetry [5] in the context of grand unification has been pointed out a long time ago [6] and recently a model for producing large L and small B has been presented [7]. Moreover, while one could naively think that the large lepton number would be washed out by the sphaleron effects at the temperature above the weak scale, it turns out that the nonrestoration of symmetry prevents this from happening [8].

Remarkably enough, having a large lepton asymmetry still remains a consistent possibility. The successful predictions of primordial nucleosynthesis are not jeopardized as long as the neutrino degeneracy parameter $\xi_\nu = \mu_\nu/T$, where μ_ν is the chemical potential of the degenerate neutrinos, is small enough. Combining the nucleosynthesis bounds [9] with the ones coming from structure formation in the Universe [10] yields $-0.06 \lesssim \xi_{\nu_e} \lesssim 1.1$ and $\left|\xi_{\nu_{\mu,\tau}}\right| \lesssim 6.9$.

It is quite intriguing that the very simple and economical hypothesis of large degeneracy in the sea of relic neutrinos may lead to so many interesting phenomena in the early Universe. If Nature has chosen the option that the lepton number is large enough so that SM symmetry (or extensions of it) is not restored at high temperature – but without any charge field condensation – the cosmological consequence would be remarkable, for this would suffice to nonrestore the symmetry. Symmetry nonrestoration provides a simple way out of the the monopole problem and the domain wall problem [11,12] which are some of the central issues in the modern astroparticle physics and are especially serious being generic to the idea of grand unification. Thus, if the lepton number of the Universe were to turn out large, there would be no monopole and domain problems whatsoever. A neutrino degeneracy ξ_ν at temperatures above 100 GeV in the range $(2.5-5.3)$ for the SM Higgs boson mass in the interval $(100-800)$ GeV would suffice to avoid the SM gauge symmetry restoration in the hot Universe [11].

Moreover, if the lepton number density of the Universe is of order of the entropy density, the neutrinos with masses in the Super-Kamiokande range ~ 0.07 eV [13] can make a significant contribution to the energy density of the Universe [14] or even explain the cosmic rays with energies in excess of the Greisen-Zatsepin-Kuzmin cutoff [15]. This would require a value of the neutrino degeneracy parameter of the order of 4.6.

The main point we wish to make, however, is that the most spectacular cosmological consequence of a large lepton asymmetry in the Universe is its impact on the temperature variations of the CMB radiation.

The CMB provides a window on fundamental physics at very high energy scales [16] and the measurement of the spectral index n, specifying the scale-dependence of the spectrum of the curvature perturbation, will be a powerful discriminator between models of inflation [17], when it is measured at the accuracy expected to result from the planned satellite missions MAP and Planck [18]. Furthermore, observations of the polarization of the CMB have the potential to place much tighter constraints on cosmological parameters than observations of the fluctuations in temperature alone. The detection

of a tensor/scalar ratio $r \sim 0.01$ would allow precision tests of interesting inflation models, and is possible with a modest increase in sensitivity over that planned for the Planck satellite, or potentially by ground-based experiments [19].

In this Letter we show that a large lepton asymmetry in the Universe leads to drastic changes in the predicted CMB anisotropies that might be unambiguously detected by future satellite experiments. This will allow us to test the presence of neutrino chemical potentials (μ_ν/T) to a precision of 1% or better. The precision increases considerably with the value of the neutrino degeneracy. This is exactly the situation one would hope for since most of the current speculations make use of large neutrino degeneracies. This, in turn, will give us an enormous amount of information about the dynamical evolution of the early Universe. Many intriguing ideas such as the possibility that the some gauge symmetries were never restored in the hot Universe because of a large lepton charge – will be tested.

In fact, the current information on the CMB anisotropy already places an upper limit on the lepton asymmetry of the Universe stronger than other considerations. We will argue that the present data disfavour values of the neutrino chemical potential (μ_ν/T) larger than about 5, independent of the neutrino flavour. The present level of knowledge about the CMB fluctuation spectrum is not only sufficient to place bounds which are more severe than the ones coming from considerations about nucleosynthesis and structure formation, but also to put meaningful bounds on some speculations about the evolution of the early Universe.

2. Lepton asymmetry and present CMB data An antisymmetry between neutrinos and antineutrinos in the universe is most conveniently measured by the chemical potential μ_ν between the two species. The difference in neutrino number density n_ν and the antineutrino number density $n_{\bar\nu}$ for a single degenerate neutrino species can be expressed as

$$n_\nu - n_{\bar\nu} = \frac{T^3}{2\pi^2}\int_{m_\nu}^{\infty} u\,du\sqrt{u^2-m^2} \times \left(\frac{1}{1+\exp\left(u-\xi_\nu\right)} - \frac{1}{1+\exp\left(u+\xi_\nu\right)}\right), \quad (1)$$

where $u \equiv E_\nu/T$, and $\xi_\nu \equiv \mu_\nu/T$. If the neutrinos are relativistic, $m_\nu \ll T$, the cosmological lepton asymmetry can be written as the ratio of the neutrino asymmetry to the entropy $L \equiv (n_\nu - n_{\bar\nu})/s = \frac{15}{4\pi^4 g_{*S}}\left(T_\nu/T_\gamma\right)^3\left(\pi^2\xi_\nu + \xi_\nu^3\right)$. The lepton asymmetry L is conserved in the cosmological expansion, and, as long as the neutrinos remain relativistic after, so is ξ. Even relatively heavy neutrinos will be relativistic until well after recombination, so for the purposes of investigating effects on the CMB, we can safely take ξ as constant. Yet, the reader should bear in mind that at temperatures larger than ~ 1 MeV $\xi_\nu \propto g_{*S}^{1/3}$ if ξ_ν is larger than unity and that our bounds refer to the present values of ξ_ν.

We will assume that the lepton asymmetry in the neutrino sector occurs in second family (μ neutrinos) or third family (τ neutrinos), so that direct effects on primordial nucleosynthesis are absent. If both neutrino families carry a chemical potential the effect on the CMB is enhanced over that for a single species. The effect of the neutrino degeneracy is then confined to: *i)* changing the time of matter/radiation equality, and *ii)* changing the time of neutrino decoupling [9]. We will further assume that the neutrinos are light enough to remain relativistic until well after recombination. This is a good approximation for neutrinos with masses in the Super-Kamiokande range.

The evaluation of the effect of the neutrino degeneracy on the CMB requires numerical evaluation of a Boltzmann equation. We use Seljak and Zaldarriaga's CMBFAST code [20] to calculate the CMB multipole spectrum, described in detail in the next section. The results are presented in Fig. 1 which shows the CMB spectrum for various values of the lepton asymmetry for a *single* family, along with the results of current CMB experiments as compiled by Tegmark [21]. Even though the current data are not conclusive in placing strict limits on the cosmological lepton asymmetry, it is evident that values of ξ_ν larger than about 5 are a poor fit to the existing data.

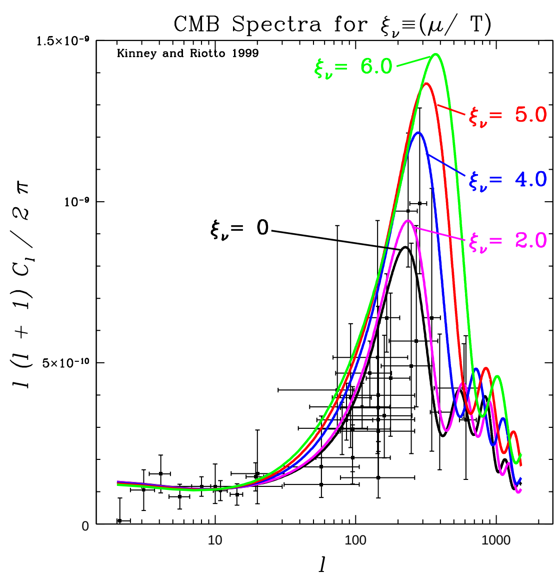

FIG. 1. CMB spectra for various $\xi_\nu \equiv (\mu_\nu/T)$, for a background cosmology with $h = 0.65$, $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. Points with error bars are currently available CMB data.

Future experiments are likely to tighten the error bars significantly. In the next section, we discuss the CMB spectrum in detail, and discuss the prospects for future experiments, particularly NASA's MAP satellite and the ESA's Planck Surveyor, to place limits on the cosmological lepton asymmetry.

3. Statistics of CMB measurements: temperature and polarization What observations of the cosmic microwave background actually measure is anisotropy in the temperature of the radiation as a function of direction. It is natural to expand the anisotropy on the sky in spherical harmonics:

$$\frac{\delta T(\theta,\phi)}{T_0} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm}^T Y_{lm}(\theta,\phi), \tag{2}$$

where $T_0 = 2.728°K$ is the mean temperature of the CMB. Inflation predicts that each a_{lm}^T will be Gaussian distributed with mean $\langle a_{lm}^T\rangle = 0$ and variance $\langle a_{l'm'}^{T*} a_{lm}^T\rangle = C_{Tl}\delta_{ll'}\delta_{mm'}$, where angle brackets indicate an average over realizations. For Gaussian fluctuations, the set of C_{Tl}'s completely characterizes the temperature anisotropy. The spectrum of the C_{Tl}'s is in turn dependent on cosmological parameters such as Ω_0, H_0, $\Omega_{\rm B}$ and so forth, so that observation of CMB temperature anisotropy can serve as an exquisitely precise probe of cosmological models.

The cosmic microwave background is also expected to be polarized due to the presence of fluctuations. Observation of polarization in the CMB will greatly increase the amount of information available for use in constraining cosmological models. Polarization is a *tensor* quantity, which can be decomposed on the celestial sphere into "electric-type", or scalar, and "magnetic-type", or pseudoscalar modes. The symmetric, trace-free polarization tensor $\mathcal{P}_{ab}$ can be expanded as [23]

$$\frac{\mathcal{P}_{ab}}{T_0} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left[a_{lm}^E Y_{(lm)ab}^E(\theta,\phi) + a_{lm}^B Y_{(lm)ab}^B(\theta,\phi)\right], \tag{3}$$

where the $Y_{(lm)ab}^{E,B}$ are electric- and magnetic-type tensor spherical harmonics, with parity $(-1)^l$ and $(-1)^{l+1}$, respectively. Unlike a temperature-only map, which is described by the single multipole spectrum of C_l^T's, a temperature/polarization map is described by three spectra

$$\left\langle\left|a_{lm}^T\right|^2\right\rangle \equiv C_{Tl},\ \left\langle\left|a_{lm}^E\right|^2\right\rangle \equiv C_{El},\ \left\langle\left|a_{lm}^B\right|^2\right\rangle \equiv C_{Bl}, \tag{4}$$

and three correlation functions, $\langle a_{lm}^{T*}a_{lm}^E\rangle \equiv C_{Cl}$, $\langle a_{lm}^{T*}a_{lm}^B\rangle \equiv C_{(TB)l}$, $\langle a_{lm}^{E*}a_{lm}^B\rangle \equiv C_{(EB)l}$. Parity requires that the last two correlation functions vanish, $C_{(TB)l} = C_{(EB)l} = 0$, leaving four spectra: temperature C_{Tl}, E-mode C_{El}, B-mode C_{Bl}, and the cross-correlation C_{Cl}. Since scalar density perturbations have no "handedness," it is impossible for scalar modes to produce B-mode (pseudoscalar) polarization. Only tensor fluctuations (or foregrounds [24]) can produce a B-mode.

Measurement uncertainty in cosmological parameters is characterized by the Fisher information matrix α_{ij}. (For a review, see Ref. [25].) Given a set of parameters $\{\lambda_i\}$, the Fisher matrix is given by

$$\alpha_{ij} = \sum_l\sum_{X,Y}\frac{\partial C_{Xl}}{\partial\lambda_i}{\rm Cov}^{-1}\left(\hat{C}_{Xl}\hat{C}_{Yl}\right)\frac{\partial C_{Yl}}{\partial\lambda_j}, \tag{5}$$

where $X,Y = T,E,B,C$ and ${\rm Cov}^{-1}\left(\hat{\rm C}_{\rm Xl}\hat{\rm C}_{\rm Yl}\right)$ is the inverse of the covariance matrix between the estimators $\hat{C}_{Xl}$ of the power spectra. Calculation of the Fisher matrix requires assuming a "true" set of parameters and numerically evaluating the C_{Xl}'s and their derivatives relative to that parameter choice. The covariance matrix for the parameters $\{\lambda_i\}$ is just the inverse of the Fisher matrix, $\left(\alpha^{-1}\right)_{ij}$, and the expected error in the parameter λ_i is of order $\sqrt{(\alpha^{-1})_{ii}}$. The full set of parameters $\{\lambda_i\}$ we allow to vary is: 1)the tensor/scalar ratio r, 2) the spectral index n, 3) the normalization $Q_{\rm rms-PS}$, 4) the baryon density $\Omega_{\rm B}$, 5) the Hubble constant $h \equiv H_0/(100\,{\rm km\,sec^{-1}\,Mpc^{-1}})$, 6) the reionization optical depth, $\tau_{\rm ri}$ and 7) the the neutrino chemical potential (μ_ν/T).

We take as a "fiducial" model COBE normalization [26] with $\Omega_{\rm B} = 0.05$ and $h = 0.65$, and a reionization optical depth of $\tau_{\rm ri} = 0.05$, corresponding to reionization at a redshift of about $z \sim 13$. The tensor/scalar ratio is $r = 0$. Fixed parameters are $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$, consistent with inflation. Assuming an approximately gaussian beam, the nonzero elements of the covariance matrix ${\rm Cov}\left(\hat{C}_{Xl}\hat{C}_{Yl}\right)$ are [27–29,23]

$$\begin{aligned}
{\rm Cov}\left(\hat{C}_{Tl}\hat{C}_{Tl}\right) &= \frac{2}{(2l+1)f_{\rm sky}}\left(C_{Tl} + w_T^{-1}e^{l^2\sigma_{\rm b}^2}\right)^2,\\
{\rm Cov}\left(\hat{C}_{El}\hat{C}_{El}\right) &= \frac{2}{(2l+1)f_{\rm sky}}\left(C_{El} + w_P^{-1}e^{l^2\sigma_{\rm b}^2}\right)^2,\\
{\rm Cov}\left(\hat{C}_{Bl}\hat{C}_{Bl}\right) &= \frac{2}{(2l+1)f_{\rm sky}}\left(C_{Bl} + w_P^{-1}e^{l^2\sigma_{\rm b}^2}\right)^2,\\
{\rm Cov}\left(\hat{C}_{Cl}\hat{C}_{Cl}\right) &= \frac{2}{(2l+1)f_{\rm sky}}\Big[C_{Cl}^2 + \\
&\quad\left(C_{Tl} + w_T^{-1}e^{l^2\sigma_{\rm b}^2}\right)\times\\
&\quad\left(C_{El} + w_P^{-1}e^{l^2\sigma_{\rm b}^2}\right)\Big],\\
{\rm Cov}\left(\hat{C}_{Tl}\hat{C}_{El}\right) &= \frac{2}{(2l+1)f_{\rm sky}}C_{Cl}^2,\\
{\rm Cov}\left(\hat{C}_{Tl}\hat{C}_{Cl}\right) &= \frac{2}{(2l+1)f_{\rm sky}}C_{Cl}\left(C_{Tl} + w_T^{-1}e^{l^2\sigma_{\rm b}^2}\right),\\
{\rm Cov}\left(\hat{C}_{El}\hat{C}_{Cl}\right) &= \frac{2}{(2l+1)f_{\rm sky}}C_{Cl}\left(C_{El} + w_P^{-1}e^{l^2\sigma_{\rm b}^2}\right).
\end{aligned} \tag{6}$$

Here $f_{\rm sky}$ is the fraction of the sky observed, and $\sigma_{\rm b} = \theta_{\rm fwhm}/\sqrt{8\ln 2}$ is the gaussian beamwidth, where $\theta_{\rm fwhm}$ is the full width at half maximum. The inverse weights per unit area w_T^{-1} and w_P^{-1} are determined by the detector resolution and sensitivity. For a noise per pixel $\sigma_{\rm pixel}^T$ and solid angle per pixel $\Omega_{\rm pixel} \simeq \theta_{\rm fwhm}^2$, the weight w_T^{-1} is

$$w_T^{-1} = \frac{\sigma_{\rm pixel}^2\Omega_{\rm pixel}}{T_0^2}. \tag{7}$$

The polarization pixel noise $\sigma_{\rm pixel}^P$ is simply related to the

temperature pixel noise $\sigma^T_{\rm pixel}$, since the number of photons available for the temperature measurement is twice that for the polarization measurements: $\left(\sigma^P_{\rm pixel}\right)^2 = 2\left(\sigma^T_{\rm pixel}\right)^2$ and $w_P^{-1} = 2w_T^{-1}$. For an observation with multiple channels c with different beam sizes and sensitivities, the weights $w_T^{(c)}$ simply add [30]. For MAP, we combine the three high-frequency channels at 40, 60, and 90 GHz, each with a pixel noise of $\sigma_{\rm pixel} = 35\,\mu$K and beam sizes $\theta_{\rm fwhm} = (28.2', 21.0', 12.6')$ respectively. Similarly, for Planck we combine the two channels at 143 and 217 GHz, with beam width $\theta_{\rm fwhm} = (8.0', 5.5')$ and pixel noise $\sigma^T_{\rm pixel} = (5.5\,\mu{\rm K}, 11.7\,\mu{\rm K})$. In all cases we take the sky fraction to be $f_{\rm sky} = 0.65$. The Table shows the expected measurement uncertainty at the 1σ level in ξ_ν for various values of the lepton asymmetry.

ξ_ν	$(\delta\xi_\nu)_{\rm MAP}$	$(\delta\xi_\nu)_{\rm Planck}$
0.25	$--$	0.10
0.5	0.4	0.05
1.0	0.2	0.02
2.0	0.09	0.01
4.0	0.04	0.005

We see that the expected measurement errors drop sharply as ξ increases, with measurement errors of order a percent possible for large ξ. MAP is capable of a marginal detection of $\xi = 0.5$, while Planck can detect ξ a factor of two smaller. What is exciting is that the uncertainties drop significantly for large values of ξ_ν, which many of the speculative proposals make use of.

4. Conclusions The lepton asymmetry of the universe is, at present, not a well-constrained quantity. In this Letter, we have shown that the cosmic microwave background is a powerful tool for placing constraints on the cosmological lepton asymmetry. In fact, the current state of knowledge about the CMB spectrum already allows useful conclusions to be drawn. (For an analysis of current constraints, see Ref. [31].) In the light of our results we may argue that the solution to the monopole problem in Grand Unified Theories as well to the domain wall problem by storing a large lepton number asymmetry in the Universe is starting to be challenged by present CMB data. The same conclusion may be drawn for the suggestion that the ultra-high energy cosmic rays beyond the Greisen-Zatsepin-Kuzmin cutoff may be explained with the aid of a neutrino degeneracy of ~ 4.6 [15].

But – luckily – the best is still to come. Future satellite experiments promise to greatly improve our knowledge of the lepton asymmetry of the Universe, with uncertainty in the chemical potential of a degenerate neutrino species of order a percent or better within reach of planned experiments. It is intriguing that future measurements of the CMB anisotropy at the expected accuracy can tell us so much about the early evolution of the Universe.

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